The importance of mathematics teachers' beliefs

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It is widely acknowledged that what teachers believe influences their teaching, yet the focus of much professional learning remains on influencing the specific practices and tools that teachers employ in their classrooms. In this article it is argued that a greater and more explicit focus on teachers' beliefs would be beneficial. To this end an overview of aspects of our understandings of the nature of beliefs is presented followed by findings from a recent study that examined mathematics teachers' beliefs and their impact on classroom practice. Finally, implications for mathematics teachers and those involved in designing and implementing professional learning for both teachers and pre-service teachers are suggested.

Belief systems

The idea of belief systems recognises that beliefs are not held in isolation from one another but are in fact inter-related in complex ways. Green (1971) provided a description of belief systems that is still very useful. He described several dimensions of beliefs systems, three of which are of relevance here. The first is the idea of centrality. The centrality of a belief is a function of the strength and number of its connections with other beliefs. Other beliefs may be held because they are consequences of a central belief and any change in a central belief would have important ramifications for the individual's belief system and could be experienced as quite unsettling. Centrally-held beliefs are thus relatively difficult to change.

A second aspect of Green's description of belief systems is the phenomenon of clustering. This means that beliefs with a system can be held in groups that are isolated from other beliefs. A consequence of this is that a person may hold beliefs that contradict one another without being aware of the contradiction. According to Green (1971) such clusters are likely to develop when beliefs are formed in disparate contexts. An example might be a student's belief that he is a poor mathematics student, formed perhaps on the basis of negative experiences of school mathematics, held at the same time as a belief in himself as mathematically competent formed as a result of experiences of part time work in a retail context. The student may not be consciously aware of one or other or both of these beliefs and may continue to believe both in the absence of any experience that makes them explicit and stimulates reflection on their contradictory elements.

The third aspect of beliefs relates to the basis on which they held. The basis of a belief may be evidence, in which case the belief is said to be evidentially held, or it may be held for other reasons such as the perceived authority of its source, or because it is regarded as a consequence of a another belief which may or may not be evidentially held. Evidentially held beliefs are by definition susceptible to change on the basis of evidence to the contrary, while non-evidentially held beliefs are impervious to evidence and hence very resistant to change.

Implicit in both the centrality and clustering of beliefs is the importance of context. The relative centrality of beliefs varies according to the context. For example, in the context of a professional learning session, a

teacher might express a belief in the importance of providing students with ready access to manipulatives as they engage with mathematics, but in the context of his grade 8 classroom his belief that the teacher must maintain control of classroom activity and the related belief that this particular class would not use manipulatives in the intended way could be more central. The result might be that manipulatives are nowhere to be seen in that classroom. It is important to recognise that this would in no way mean that there was any lack of sincerity associated with the teacher's statement during the professional learning session.

The notion of clustering provides an alternative explanation for apparent contradictions between stated beliefs and practices like that described above. It allows the possibility that a teacher might simultaneously hold contradictory beliefs that have developed in different contexts. Beliefs formed as result of his/her own experiences of learning mathematics, those formed during teacher education, and others that have developed as result of classroom experience may contain contradictory elements that the teacher is unaware of.

The study

The study aimed to examine the connection between secondary mathematics teachers' beliefs and their mathematics classroom environments and was described in detail by Beswick (2005, in press). It involved surveys of 25 teachers and 39 of their mathematics classes as well as interviews with eight of the teachers and observations of approximately 12 lessons taught by each of six of the interviewed teachers. The first teacher survey asked them to indicate the extent of their agreement with 26 statements about the nature of mathematics, mathematics teaching and mathematics learning. A second survey, the Classroom Learning Environment Survey (CLES) (Taylor, Fraser & Fisher, 1993) sought their perceptions of their classroom environments and asked them to rate the frequency of occurrence of various classroom events. The teachers were asked to complete this survey twice with a particular mathematics class in mind on each occasion. Several of the teachers completed just one survey either because they believed that the classroom environments of both classes were the same or because of time constraints. The teachers administered a student version of the CLES to the students in the two classes with respect to which they had completed the teacher version.

The vast majority ($\geq$ 88%) of the teachers agreed or strongly agreed with statements such as the following:

1. A vital task for the teacher is motivating children to solve their own mathematical problems.
2. Ignoring the mathematical ideas that children generate themselves can seriously limit their learning.
3. It is important for children to be given opportunities to reflect on and evaluate their own mathematical understanding.
4. It is important for teachers to understand the structured way in which mathematics concepts and skills relate to each other.
5. Effective mathematics teachers enjoy learning and "doing" mathematics themselves.
6. Knowing how to solve a mathematics problem is as important as getting the correct solution.
7. Teachers of mathematics should be fascinated with how children think and intrigued by alternative ideas.
8. Providing children with interesting problems to investigate in small groups is an effective way to teach mathematics

It is important to note that, with the exception of number 8, none of these statements prescribe any particular teaching strategy or classroom arrangement. The teachers were less inclined to agree with statements that did. For example, less than two-thirds of the teachers agreed or strongly agreed with the following items:

9. It is the teacher's responsibility to provide children with clear and concise solution methods for mathematical problems.
10. There is an established amount of mathematical content that should be covered at each grade level.

11. It is important that mathematics content be presented to children in the correct sequence.
12. Mathematical material is best presented in an expository style: demonstrating, explaining and describing concepts and skills.

Number 12 more evenly divided the teachers than any other (32% agreed or strongly agreed, 40 % undecided, 28% disagreed or strongly disagreed) indicating a diversity of opinion, as well as considerable uncertainty, regarding how beliefs such as those expressed in statements 1–8 should be enacted.

Cluster analysis (Hair, Anderson, Tatham & Black, 1998) was used to group the 25 into three clusters according to their responses to the beliefs survey and also to group the students according to their responses to the CLES (student version) (Beswick, 2005). The beliefs survey resulted in three clusters of teachers. These were:

1. **Content and clarity**
   These teachers believed that they had a responsibility clearly to explain mathematical content and that it may be necessary to tell students the answers. They believed that they must cover the prescribed content in the correct sequence. They also regarded computation is a major part of mathematics and believed that effective mathematics teachers enjoyed the discipline.

2. **Relaxed problem solvers**
   Teachers in this cluster viewed mathematics as more than computation and were the least inclined to believe that it was their role to provide answers or even clear solution methods. They were also less concerned than other teachers about either content coverage or sequencing.

3. **Content and understanding**
   These teachers could be described as the most concerned about the coverage and sequencing of the content, but the least likely to seek guidance regarding sequencing from a textbook. They were focussed on students' understanding of the content, but not comfortable with students suggesting alternative solutions.

The CLES (student version) resulted in five clusters of classes based on the classes' average perceptions of the extent to which they were responsible for their learning and were engaged with the mathematics and connecting their learning with their existing knowledge (Beswick, 2005). The more these elements were in evidence the more consistent with constructivist principles the classrooms were deemed to be.

Subtle but important relationships were found between the teacher's beliefs and their students' average perceptions of their classroom environments (Beswick, 2005). Classes in clusters characterised by classroom environments most consistent with constructivist principles were more likely than others to be taught by teachers whose belief survey responses placed them in the Relaxed Problem Solvers cluster. It is important to remember, however, that teachers in this cluster (and each of the others) did not achieve these classroom environments by implementing identical, or even superficially similar, practices, but in spite of the variety of ways in which they were implemented, their beliefs impacted their classrooms in ways that their students could discern. This fact is illustrated by two of the teachers in the Relaxed problem solvers cluster, Jim and Andrew (pseudonyms), who were also interviewed and observed in their classrooms. Both of these teachers had classes in the cluster characterised by classroom environments most consistent with constructivist principles.

The following quotations, some of which also appear in Beswick (in press), are taken from the interviews with Jim and Andrew and provide an indication of their beliefs about the discipline of mathematics and mathematics teaching and learning.

Jim

> I read about it, and I enjoy it and I sit here folding bits of paper in times when I could be doing something adults think might be more important… and I'm constantly excited by it, and I do a fair bit of personal professional

development and every time I go somewhere, I find extra little things… (Beswick, in press)

[I]f we're investigating some aspect of that, and the kids come up with a "what if" idea or, "I wonder what happens if we do this", then I'd absolutely grab it all the time… If you think you've planned this lesson, and it's beautiful and linear and, it's going to work… but I think the kids sometimes, won't believe they've got anything to offer… and if we're going to keep inventing this stuff called mathematics, or discovering it, or making sense of it, we've got to believe some of our kids are going to go a lot further than we did, and if they don't think they can actually offer anything they won't.

Sometimes when kids have suggestions, they're incomprehensible, if you just listen to their words because they haven't got the language and they haven't got the background… and it's easy to dismiss stuff as being ludicrous, but if you then have got a culture where they can sit and try and tease it out and explain it, often they come up with amazing sorts of things…

Andrew

I'm very teacher directed but at the same time what I like to do is not to give the kids the answers, but what I try to do is to make them think… Getting them to come up and put on the board their ideas, what they think might be, what they should be doing or their way of doing something is a struggle…

Yes, they should guess. They should conjecture, but at some stage the teacher's going to have to call a halt and just say well, what about trying this? … You're not just a supporting role, you are a facilitator, but you're also more than that. You're someone who hopefully understands the clear path that might be needed and can also see different paths to get to the end point and send the kids off on appropriate paths, not just let them wander through the minefield.

Observations of Jim's classes (grades 9 and 10) and Andrew's grade 7 classes confirmed their interview responses and revealed very different teaching approaches at least superficially. For example, Jim's students almost always worked in small groups and his interactions with them were primarily at the individual or small group level. In contrast with this, Andrew's students sat in rows of twos or threes facing the front of the room and most of the interactions were at the level of whole class discussions facilitated by Andrew. Nevertheless, the students perceptions of their classroom environments indicated that there were similarities in the extent to which they were responsible for their learning and were engaged with the mathematics and connecting their learning with their existing knowledge.

The beliefs that emerged as underpinning the practice of Jim and Andrew related to the nature of mathematics, their students and their capabilities, the teacher's role in the classroom and professional learning. Beliefs about mathematics, students, and the importance of professional learning were most central in Jim's case, whereas beliefs about the teacher's role were most central for Andrew. The particular beliefs that emerged as most central to one or other of Jim and Andrew were:

1. Mathematics is about connecting ideas and sense-making.
2. Mathematics is fun (in the sense of playful confidence with and enjoyment of mathematics).
3. Students' learning is unpredictable.
4. All students can learn mathematics.
5. The teacher has a responsibility to maintain ultimate control of the classroom discourse.
6. The teacher has a responsibility actively to facilitate and guide students' construction of mathematical knowledge.
7. The teacher has a responsibility to induct students into widely accepted ways of thinking and communicating in mathematics.
8. The teacher is the authority with respect to the social norms that operate in the classroom.
9. Teachers have a professional responsibility to engage in ongoing learning.

Beswick (in press) argued that this set of beliefs seems to be related to teachers' ability to create classroom environments that can be described as constructivist and that it is such beliefs, rather than particular teaching methods or materials, that matter in terms of students' perceptions of their classroom environments. This is consistent with the findings of Watson and De Geest (2005) and Askew, Brown, Rhodes, Johnson and Wiliam (1997) concerning the importance of teachers' beliefs in shaping their practices.

Implications

The literature on teacher change is replete with evidence that real and lasting change is achieved only if teachers' belief systems support the underlying premises of the changes they are asked to implement (e.g., Chapman, 2002). Little is achieved by getting teachers (or students) to mouth "suitable" views or perform certain actions if they are not convinced of their value. It is, therefore, not enough to provide teachers with resources, curriculum materials and ideas without attending to their relevant beliefs. The point here is analogous to the more widely espoused view that it is not enough to get students to recite facts or perform procedures if they are not meaningful to them — i.e., if they do not really believe the procedures or their results.

Findings concerning the importance of teachers' beliefs to the kinds of classrooms that they create highlight the importance of individual mathematics teachers, and providers of professional learning or preservice teacher education related to mathematics, reflecting carefully on the beliefs that they hold about the nature of mathematics and about mathematics teaching and learning. The following is a list of questions that may be helpful in stimulating such reflection:

With respect to each of the nine beliefs listed above:

1. To what extent do I hold this belief?
2. Why do I believe this? What hard evidence underpins my belief? Is this evidence more than anecdote?
3. How/in what way(s) does this belief shape my practice?
4. How would my practice be different if I believed this?
5. Would an observer in my class (including my students) be surprised if I told them I believed this? Why?
6. What other beliefs about mathematics or mathematics teaching and learning, influence my practice? Why do I believe these things? Is there hard evidence for their veracity?

When considering new practices, ideas, or materials:

7. What beliefs about mathematics and about mathematics teaching and learning does the author/creator of these materials hold?
8. What does this professional learning provider believe about the nature of mathematics and mathematics learning and teaching?
9. To what extent do I share these beliefs? Why?
10. What beliefs underpin my negative/positive reaction to this idea? Are these beliefs reasonable?

In relation to students' perceptions of your beliefs:

11. What might my students think I believe about:
    a. their capacity to learn mathematics?
    b. how they learn mathematics?
    c. what it means to "do mathematics"?
    d. my role as a mathematics teacher?
12. How might these perceptions vary from student to student or from class to class? Would there be differences according to mathematical ability or grade level? How might students notice these differences?

Opportunities to talk with trusted colleagues about responses to these questions would likely be helpful. It would also seem sensible for professional learning providers to be explicit about their own beliefs and those that underpin their own practices and recommendations. Providing time, opportunities and stimuli for teachers' reflection on their beliefs

is also important and certainly consistent with a social constructivist view of learning that recognises that teacher change is learning. Similarly teachers should make their own relevant beliefs explicit for their students. Perhaps teachers and teacher educators alike could benefit from asking their "students" what they think their "teachers" believe. All of this has the potential to be quite confronting and uncomfortable but I believe that such unsettling is fundamental to learning.

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Diversions

Solutions

1. Capture

a. 12, to avoid being captured.
b. No. You will never be able to capture Player 1 without moving to 00 and returning to a suitable position.
c. 11, 31 or 51 as this forces Player 2 to move to position 00 before being able to capture you.
d. and e. I think I have given enough hints.

2. Take away

a. Take away 2
b. Take away 1 or 3
c. Take away 9
d. and e. I think I have given enough hints.